Parametric Response of an Axially Moving Viscoelastic Beam With Three-Mode Interaction

Usama H. Hegazy

Abstract—The response of axially moving 3D-beam, supported by nonlinear viscoelastic foundation under parametric excitation, is discussed and the governing nonlinear partial differential equation of motion is discretized into ordinary differential equations using 3-term Galerkin method. The approximate solutions are obtained applying the multiple scales perturbation technique and the case of external subharmonic resonance and 1:1:1 internal resonance are considered. The effects of viscoelastic coefficient, nonlinear coefficients, stiffness, and axial moving speed as well as the magnitude of the parametric excitation on the frequency responses are investigated.

Keywords: Nonlinear oscillations, Multiple scales method, Subharmonic resonance, Viscoelastic beam, Frequency response curves

1. Introduction

Yang et al. [1] considered axially moving viscoelastic beam under multi-frequency excitations and used 1-term Galerkin method to reduce the derived nonlinear partial differential equation of motion to ordinary equation. The method of multiple scales method is utilized to obtain the approximate solutions of the transverse vibration of the beam which are compared with numerical results. Oz and Pakdermirli [2] and Ozkaya and Pakdermirli [3] studied the dynamic vibration of an axially moving beam with time-dependent velocity and with small flexural stiffness. The multiple scales method is applied directly to the nonlinear partial differential equation of motion. Yang and Chen [4] derived the governing partial differential equation of motion of an axially accelerating beam, based on the Kelvin-Voigt model. The Galerkin method is applied to obtain two dimensional ordinary differential equations. Poincare maps, the phase plane and the largest Lyapunov exponent are employed to investigate numerically the nonlinear dynamic response of the model with time-varying transport speed. The effects of the mean axial speed, the amplitude of the speed fluctuation and the dynamic viscoelasticity are shown using bifurcation diagrams. Argyris et al. [5] investigated the chaotic and regular responses of a nonlinear viscoelastic beam subjected to a periodically forced excitation using the Poincare mapping and the Lyapunov exponent techniques. It is shown that the system possesses a very complex fractal geometry when the basins of attraction are plotted. Adetunde and Seidu [6] studied the dynamic behavior of a viscously damped Rayleigh beam to axial force and obtained the numerical solution of the governing differential equation by the finite central difference method. It is found that the mass of the moving load depends on the deflection along the length of the beam and that the deflection of the moving mass is greater than that of the moving force [6]. Xiao-dong and Li-qun [7] used the averaging method to study the stability of an axially moving beam with pulsating speed at subharmonic and combination resonances and verified the analytical results numerically. It is shown that as the steady speed or the compression tension is increased the instability region gets larger, whereas increasing the viscoelastic damping leads to a small instability region. Lee and Oh [8] developed a spectral element model for an axially moving viscoelastic beam subject to axial tension and investigated the effects of moving speed and viscoelasticity on the stability of the system. It is found that in the case of pure elastic moving beam, a single coupled-mode flutter may occur, while for the case of viscoelastic moving beam, only first natural mode becomes unstable with flutter. Ansari et al. [9] used the Galerkin method to reduce the governing partial differential equation of a nonlinear viscoelastic beam under moving load to three dimensional nonlinear ordinary differential equations. The multiple scales perturbation technique is utilized to investigate the responses of the three modes of the vibrating system considering several internal-external resonance cases. The effects of different parameters on the system behavior are also studied. Chen et al. [10] investigated the nonplanar nonlinear chaotic oscillations and bifurcations of an axially accelerating moving viscoelastic beam. A three and six ordinary differential governing equations with parametric excitations are obtained from the established governing equations of motion, using the generalized Hamilton's principle. Numerical investigations are performed using the Poincare maps, the phase planes and the largest Lyapunov exponents. It is shown that the three and the six d.o.f systems have very different nonlinear dynamic responses.

In this paper, the nonlinear dynamic response of an axially moving 3-D beam on elastic foundation is investigated. The Galerkin method is used to discretize the governing nonlinear partial differential equation of the beam under parametric excitation. The method of multiple scales is utilized and the case in which there simultaneously exist principal parametric resonance and primary internal resonance for the three modes of the vibrating beam is considered. The steady-state responses are studied by solving numerically the resonant frequency response equations. A comprehensive numerical investigation is carried out to show the effect of different parameters on the dynamic performance of each mode of vibration.

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2. Problem formulation

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The nondimensional equation governing the transverse motion of an axially moving viscoelastic beam has been modified as follows [1]:

$$\frac{\partial^2 u}{\partial t^2}+2c\frac{\partial^2 u}{\partial x\,\partial t}+(c^2-1)\frac{\partial^2 u}{\partial x^2}+\beta^2\frac{\partial^4 u}{\partial x^4}$$
$$=\alpha\frac{\partial^5 u}{\partial x^2\partial t}+\frac{1}{2}\delta^2\frac{\partial^2 u}{\partial x^2}\int_0^1\left(\frac{\partial u}{\partial x}\right)^2 dx+uF(x,t) \qquad (1)$$

The beam is assumed simply supported at both ends with the boundary conditions

$$u(0,t)=u(1,t)=0;\ \frac{\partial^2 u(0,t)}{\partial x^2}=\frac{\partial^2 u(1,t)}{\partial x^2}=0 \qquad (2)$$

where u is the transverse displacement, c is the axial constant velocity, α is the viscoelastic coefficient, β is the flexural stiffness, δ is a nonlinear coefficient and F is the parametric excitation. These parameters can be found in ref. [1]. The Galerkin method is applied to discretize the governing partial differential equation of the model (1) with the following expansion in u

$$u(x,\ t)=\sum_{n=1}^{\infty}q_n(t)\sin(n\pi x) \qquad (3)$$

where $q(t)$ is the amplitude of the fundamental transverse mode and setting n = 3, n denotes number of modes of vibrations, results in three degrees of freedom equations. Substituting equation (3) in equation (1), applying the principle of orthogonality of mode shapes and multiplying the resulting equation by sin(nπx) then performing integration over the interval [0, 1] gives:

$$\ddot{q}_1+\omega_1^2 q_1=\varepsilon\big(-\mu_1\dot{q}_1+\mu_4\dot{q}_2-\alpha_1 q_1^3-\alpha_2 q_1 q_2^2$$
$$-\alpha_3 q_1 q_3^2+q_1 F\cos(\Omega_1 t+\tau_1)\big), \qquad (4)$$

$$\ddot{q}_2+\omega_2^2 q_2=\varepsilon\big(-\mu_2\dot{q}_2-\mu_4\dot{q}_1+\mu_5\dot{q}_3-\alpha_4 q_2^3$$
$$-\alpha_5 q_2 q_1^2-\alpha_6 q_2 q_3^2+q_2 G\cos(\Omega_2 t+\tau_2)\big), \qquad (5)$$

$$\ddot{q}_3+\omega_3^2 q_3=\varepsilon\big(-\mu_3\dot{q}_3-\mu_5\dot{q}_2-\alpha_7 q_3^3-\alpha_8 q_3 q_1^2$$
$$-\alpha_9 q_3 q_2^2+q_3 H\cos(\Omega_3 t+\tau_3)\big). \qquad (6)$$

where $\omega_k^2=k^2(1-c^2)\pi^2+k^4(\beta^2\pi^4)$, k = 1,2,3 are the natural frequencies of the three modes, $\alpha_1=\frac{1}{4}\pi^4\delta$, $\alpha_2=\alpha_5=4\alpha_1$, $\alpha_3=\alpha_8=9\alpha_1$, $\alpha_4=4\alpha_2$, $\alpha_6=\alpha_9=9\alpha_2$, $\alpha_7=81\alpha_2$ are nonlinear coefficients. $\mu_1=\pi^4\alpha$, $\mu_2=16\mu_1$, $\mu_3=81\mu_1$, $\mu_4=\frac{16}{3}c$, $\mu_5=\frac{9}{5}\mu_4$ are the damping coefficients. F, G, H and Ω_j (j =1,2,3) are the forcing amplitudes and frequencies, τ_j (j =1,2,3) are constants.

3. Perturbation Analysis

In equations (4-6), ε is assumed to be a small dimensionless bookkeeping parameter. The method of multiple scales is utilized to determine a first-order uniform expansion for the solution of equations (4-6) in the form.

$$q_j(t,\ \varepsilon)=q_{j0}(T_0,\ T_1)+\varepsilon q_{j1}(T_0,\ T_1)+O(\varepsilon^2),\ j=1,2,3 \qquad (7)$$

where, $T_n=\varepsilon^n t$ and $T_0=t$.

By substituting equation (7) into equations (4-6) and equating the coefficients of the same powers of ε we obtain the following differential equations

Order ε^0 : $(D_0^2+\omega_j^2)q_{j0}=0$, $j=1,2,3$ (8)

Order ε^1 :

$$(D_0^2+\omega_1^2)q_{11}=-2D_0D_1q_{10}-\mu_1D_0q_{10}$$
$$+\mu_4D_0q_{20}-\alpha_1q_{10}^3-\alpha_2q_{10}q_{20}^2$$
$$-\alpha_3q_{10}q_{30}^2+F\cos(\Omega_1t+\tau_1), \qquad (9)$$

$$(D_0^2+\omega_2^2)q_{21}=-2D_0D_1q_{20}-\mu_2D_0q_{20}$$
$$+\mu_5D_0q_{30}-\mu_4D_0q_{10}-\alpha_4q_{20}^3$$
$$-\alpha_5q_{20}q_{10}^2-\alpha_6q_{20}q_{30}^2$$
$$+G\cos(\Omega_2t+\tau_2), \qquad (10)$$

$$(D_0^2+\omega_3^2)q_{31}=-2D_0D_1q_{30}-\mu_3D_0q_{30}$$
$$-\mu_4D_0q_{20}-\alpha_7q_{30}^3-\alpha_8q_{30}q_{10}^2$$
$$-\alpha_9q_{30}q_{20}^2+H\cos(\Omega_3t+\tau_3). \qquad (11)$$

The solution of equation (8) can be written as

$$q_{j0}(T_0,T_1)=A_j(T_1)\exp(i\,\omega_jT_0)+cc,\ j=1,2,3 \qquad (12)$$

where A_j are complex functions in T_1 (cc denotes a complex conjugate of the preceding terms). Substituting equation (12) into equations (9-11), we get

$$(D_0^2+\omega_1^2)q_{11}=[-i\,\omega_1(2A_1'+\mu_1A_1)+i\,\mu_4\omega_2A_2-3\alpha_1A_1^2\bar{A}_1-2\alpha_2A_1A_2\bar{A}_2$$
$$-2\alpha_3A_1A_3\bar{A}_3]\exp(i\omega_1T_0)\ -\alpha_1A_1^3\exp(3i\omega_1T_0)$$
$$-\alpha_2A_1A_2^2\exp(i(\omega_1+2\omega_2)T_0)$$
$$-\alpha_2A_1\bar{A}_2^2\exp(-i(\omega_1-2\omega_2)T_0)$$
$$-\alpha_3A_1A_3^2\exp(-i(\omega_1+2\omega_3)T_0)$$
$$-\alpha_3A_1\bar{A}_3^2\exp(-i(\omega_1-2\omega_3)T_0)$$
$$+\frac{F}{2}[A_1\exp(i\,(\Omega_1+\omega_1)T_0+\tau_1)+\bar{A}_1\exp(i\,(\Omega_1-\omega_1)T_0+\tau_1)]+cc, \qquad (13)$$

$$(D_0^2+\omega_2^2)q_{21}=[-i\,\omega_2(2A_2'+\mu_2A_2)+i\,\mu_5\omega_3A_3-i\,\mu_4\omega_1A_1-3\alpha_4A_2^2\bar{A}_2$$

$$
\begin{aligned}
&-2\alpha_5 A_1\bar{A}_1A_2 - 2\alpha_6 A_2A_3\bar{A}_3]\exp(i\,\omega_2T_0)\\
&-\alpha_4A_2^3\exp(3i\omega_2T_0)\\
&-\alpha_5A_2A_1^2\exp(i(\omega_2+2\omega_1)T_0)\\
&-\alpha_5A_2\bar{A}_1^2\exp(-i(\omega_2-2\omega_1)T_0)\\
&-\alpha_6A_2A_3^2\exp(-i(\omega_2+2\omega_3)T_0)\\
&-\alpha_6A_2\bar{A}_3^2\exp(-i(\omega_2-2\omega_3)T_0)\\
&+\frac{G}{2}[A_2\exp(i\,(\Omega_2+\omega_2)T_0+\tau_2)\\
&+\bar{A}_2\exp(i\,(\Omega_2-\omega_2)T_0+\tau_2)]+cc, \qquad (14)
\end{aligned}
$$

$$
\begin{aligned}
(D_0^2+\omega_3^2)q_{31} =& [-i\,\omega_3(2A_3'+\mu_3A_3)-i\,\mu_4\omega_2A_2-3\alpha_7A_3^2\bar{A}_3\\
&-2\alpha_8A_3A_1\bar{A}_1-2\alpha_9A_3A_2\bar{A}_2]\exp(i\,\omega_3T_0)\\
&-\alpha_7A_3^3\exp(3i\omega_3T_0)\\
&-\alpha_8A_3A_1^2\exp(i(\omega_3+2\omega_1)T_0)\\
&-\alpha_8A_3\bar{A}_1^2\exp(-i(\omega_3-2\omega_1)T_0)\\
&-\alpha_9A_3A_2^2\exp(-i\,(\omega_3+2\omega_2)T_0)\\
&-\alpha_9A_3\bar{A}_2^2\exp(-i(\omega_3-2\omega_2)T_0)\\
&+\frac{H}{2}[A_3\exp(i\,(\Omega_3+\omega_3)T_0+\tau_3)\\
&+\bar{A}_1\exp(i\,(\Omega_3-\omega_3)T_0+\tau_3)]+cc. \qquad (15)
\end{aligned}
$$

where the prime indicates the derivative with respect to T_1.

3.1 Principal Parametric and Primary Internal Resonances

The case of principal parametric (subharmonic) resonance will be considered for the three modes of vibrations simultaneously. The considered resonance case occurs when $\Omega_1 \approx 2\omega_1$, $\Omega_2 \approx 2\omega_2$ and $\Omega_3 \approx 2\omega_3$ in the presence of primary internal resonance between three modes of vibration ($\omega_1 = \omega_2 = \omega_3$). The closeness of the resonances is described by introducing the external detuning parameters σ_1, σ_2, and σ_3 as

$$\Omega_1 = 2\omega_1 + \varepsilon\,\sigma_1,\ \ \Omega_2 = 2\omega_2 + \varepsilon\,\sigma_2 \text{ and } \Omega_3 = 2\omega_3 + \varepsilon\,\sigma_3 . \qquad (16)$$

Using (16) in eliminating terms that produce secular terms from equations (13-15), gives the solvability conditions as

$$
\begin{aligned}
&-2i\omega_1A_1'-i\mu_1\omega_1A_1+i\mu_4\omega_2A_2-3\alpha_1A_1^2\bar{A}_1-2\alpha_2A_1A_2\bar{A}_2-2\alpha_3A_1A_3\bar{A}_3\\
&-\alpha_2\bar{A}_1A_2^2\exp(i(\sigma_2-\sigma_1)T_1)\ -\alpha_3\bar{A}_1A_3^2\exp(i(\sigma_3-\sigma_1)T_1)\\
&+\frac{1}{2}F\,\bar{A}_1\exp(i\,\sigma_1T_1)=0, \qquad (17)
\end{aligned}
$$

$$
\begin{aligned}
&-2i\,\omega_2A_2'-i\,\mu_2\omega_2A_2+i\,\mu_5\omega_3A_3-i\,\mu_4\omega_1A_1-3\alpha_4A_2^2\bar{A}_2-2\alpha_5A_1\bar{A}_1A_2\\
&-2\alpha_6A_2A_3\bar{A}_3-\alpha_5A_1^2\bar{A}_2\exp(i\,(\sigma_1-\sigma_2)T_1)\\
&-\alpha_6\bar{A}_2A_3^2\exp(i\,(\sigma_3-\sigma_2)T_1)+\frac{1}{2}G\,\bar{A}_2\exp(i\,\sigma_2T_1)=0, \qquad (18)
\end{aligned}
$$

$$
\begin{aligned}
&-2i\omega_3A_3'-i\mu_3\omega_3A_3-i\mu_5\omega_2A_2-3\alpha_7A_3^2\bar{A}_3-2\alpha_8A_1\bar{A}_1A_3-2\alpha_9A_2\bar{A}_2A_3\\
&-\alpha_8A_1^2\bar{A}_3\exp(i\,(\sigma_1-\sigma_3)T_1)-\alpha_9A_2^2\bar{A}_3\exp(i\,(\sigma_2-\sigma_3)T_1)\\
&+\frac{1}{2}H\,\bar{A}_3\exp(i\,\sigma_3T_1)=0. \qquad (19)
\end{aligned}
$$

Substituting the polar coordinates notation

$$A_k = \frac{1}{2}a_k\,\exp(i\,\theta_k),\ k=1,2,3 \qquad (20)$$

into equations (17-19), where a_k and θ_k are the steady-state amplitudes and the phases of the motions respectively, then separating the real and imaginary parts gives the governing equations of the amplitudes a_k and phases γ_i

$$
\begin{aligned}
a_1' =& -\frac{1}{2}\mu_1a_1-\frac{\omega_2}{2\omega_1}\mu_4a_2\cos\theta-\frac{\alpha_2}{8\omega_1}a_1a_2^2\sin\gamma_2\\
&-\frac{\alpha_3}{8\omega_1}a_1a_3^2\sin\gamma_3+\frac{Fa_1}{4\omega_1}\sin\gamma_1, \qquad (21)
\end{aligned}
$$

$$
\begin{aligned}
a_1\gamma_1' =& \ \sigma_1a_1-\frac{3\alpha_1}{4\omega_1}a_1^3-\frac{\alpha_2}{2\omega_1}a_1a_2^2-\frac{\alpha_3}{2\omega_1}a_1a_3^2\\
&-\frac{\omega_2}{\omega_1}\mu_4a_2\sin\theta-\frac{\alpha_2}{4\omega_1}a_1a_2^2\cos\gamma_2\\
&-\frac{\alpha_3}{4\omega_1}a_1a_3^2\cos\gamma_3+\frac{Fa_1}{2\omega_1}\cos\gamma_1, \qquad (22)
\end{aligned}
$$

$$
\begin{aligned}
a_2' =& -\frac{1}{2}\mu_2a_2-\frac{\omega_3}{2\omega_2}\mu_5a_3\cos\theta_4\\
&-\frac{\omega_1}{2\omega_2}\mu_4a_1\cos\theta_5-\frac{\alpha_5}{8\omega_2}a_2a_1^2\sin\gamma_6\\
&-\frac{\alpha_6}{8\omega_2}a_2a_3^2\sin\gamma_5+\frac{Ga_2}{4\omega_2}\sin\gamma_4, \qquad (23)
\end{aligned}
$$

$$
\begin{aligned}
a_2\gamma_4' =& \ \sigma_2a_2-\frac{3\alpha_4}{4\omega_2}a_2^3-\frac{\alpha_5}{2\omega_2}a_2a_1^2-\frac{\alpha_6}{2\omega_2}a_2a_3^2\\
&-\frac{\omega_3}{\omega_2}\mu_5a_3\sin\theta_4-\frac{\omega_1}{\omega_2}\mu_4a_1\sin\theta_5\\
&-\frac{\alpha_5}{4\omega_2}a_2a_1^2\cos\gamma_6-\frac{\alpha_6}{4\omega_2}a_2a_1^2\cos\gamma_5+\frac{Ga_2}{2\omega_2}\cos\gamma_4, \qquad (24)
\end{aligned}
$$

$$
\begin{aligned}
a_3' =& -\frac{1}{2}\mu_3a_3-\frac{\omega_2}{2\omega_3}\mu_5a_2\cos\theta_6-\frac{\alpha_8}{8\omega_3}a_3a_1^2\sin\gamma_9\\
&-\frac{\alpha_9}{8\omega_3}a_3a_2^2\sin\gamma_8+\frac{Ha_3}{4\omega_3}\sin\gamma_7, \qquad (25)
\end{aligned}
$$

$$\begin{aligned} a_3\gamma_7' &= \sigma_3 a_3 - \frac{3\alpha_7}{4\omega_3}a_3^3 - \frac{\alpha_8}{2\omega_3}a_3a_1^2 - \frac{\alpha_9}{2\omega_3}a_3a_2^2 \\ &\quad - \frac{\omega_2}{\omega_3}\mu_5 a_3\sin\theta_6 - \frac{\alpha_8}{4\omega_3}a_3a_1^2\cos\gamma_9 \\ &\quad - \frac{\alpha_9}{4\omega_3}a_3a_2^2\cos\gamma_8 + \frac{Ha_1}{2\omega_3}\cos\gamma_7. \end{aligned} \qquad (26)$$

where
$\theta = \theta_2 - \theta_1$, $\theta_4 = \theta_3 - \theta_2$, $\theta_5 = \theta_1 - \theta_2$, $\theta_6 = \theta_2 - \theta_3$, $\gamma_1 = \sigma_1 T_1 - 2\theta_1$, $\gamma_2 = 2(\theta_2 - \theta_1) - (\sigma_2 - \sigma_1)T_1$, $\gamma_3 = 2(\theta_3 - \theta_1) - (\sigma_3 - \sigma_1)T_1$, $\gamma_4 = \sigma_2 T_1 - 2\theta_2$, $\gamma_5 = 2(\theta_3 - \theta_2) - (\sigma_3 - \sigma_2)T_1$, $\gamma_6 = 2(\theta_1 - \theta_2) - (\sigma_1 - \sigma_2)T_1$, $\gamma_7 = \sigma_3 T_1 - 2\theta_3$, $\gamma_8 = 2(\theta_2 - \theta_3) - (\sigma_2 - \sigma_3)T_1$, $\gamma_9 = 2(\theta_1 - \theta_3) - (\sigma_1 - \sigma_3)T_1$.

The above equations (21-26) are solved numerically to find the steady-state responses of the three modes of vibrations, which correspond to constant solutions, that is correspond to $a'_{1,2,3} = 0$ and $\gamma'_{1,4,7} = 0$. Hence the fixed points of equations (21-26) are given by

$$\begin{aligned} \frac{1}{2}\mu_1 a_1 &= -\frac{\omega_2}{2\omega_1}\mu_4 a_2\cos\theta - \frac{\alpha_2}{8\omega_1}a_1a_2^2\sin\gamma_2 \\ &\quad - \frac{\alpha_3}{8\omega_1}a_1a_3^2\sin\gamma_3 + \frac{Fa_1}{4\omega_1}\sin\gamma_1, \end{aligned} \qquad (27)$$

$$\begin{aligned} \sigma_1 a_1 &= \frac{3\alpha_1}{4\omega_1}a_1^3 + \frac{\alpha_2}{2\omega_1}a_1a_2^2 + \frac{\alpha_3}{2\omega_1}a_1a_3^2 \\ &\quad + \frac{\omega_2}{\omega_1}\mu_4 a_2\sin\theta + \frac{\alpha_2}{4\omega_1}a_1a_2^2\cos\gamma_2 \\ &\quad + \frac{\alpha_3}{4\omega_1}a_1a_3^2\cos\gamma_3 - \frac{Fa_1}{2\omega_1}\cos\gamma_1, \end{aligned} \qquad (28)$$

$$\begin{aligned} \frac{1}{2}\mu_2 a_2 &= -\frac{\omega_3}{2\omega_2}\mu_5 a_3\cos\theta_4 - \frac{\omega_1}{2\omega_2}\mu_4 a_1\cos\theta_5 \\ &\quad - \frac{\alpha_5}{8\omega_2}a_2a_1^2\sin\gamma_6 - \frac{\alpha_6}{8\omega_2}a_2a_3^2\sin\gamma_5 \\ &\quad + \frac{Ga_2}{4\omega_2}\sin\gamma_4, \end{aligned} \qquad (29)$$

$$\begin{aligned} \sigma_2 a_2 &= \frac{3\alpha_4}{4\omega_2}a_2^3 + \frac{\alpha_5}{2\omega_2}a_2a_1^2 + \frac{\alpha_6}{2\omega_2}a_2a_3^2 + \frac{\omega_3}{\omega_2}\mu_5 a_3\sin\theta_4 \\ &\quad + \frac{\omega_1}{\omega_2}\mu_4 a_1\sin\theta_5 + \frac{\alpha_5}{4\omega_2}a_2a_1^2\cos\gamma_6 + \frac{\alpha_6}{4\omega_2}a_2a_1^2\cos\gamma_5 \\ &\quad - \frac{Ga_2}{2\omega_2}\cos\gamma_4, \end{aligned} \qquad (30)$$

$$\begin{aligned} \frac{1}{2}\mu_3 a_3 &= -\frac{\omega_2}{2\omega_3}\mu_5 a_2\cos\theta_6 - \frac{\alpha_8}{8\omega_3}a_3a_1^2\sin\gamma_9 \\ &\quad - \frac{\alpha_9}{8\omega_3}a_3a_2^2\sin\gamma_8 + \frac{Ha_3}{4\omega_3}\sin\gamma_7, \end{aligned} \qquad (31)$$

$$\begin{aligned} \sigma_3 a_3 &= \frac{3\alpha_7}{4\omega_3}a_3^3 + \frac{\alpha_8}{2\omega_3}a_3a_1^2 + \frac{\alpha_9}{2\omega_3}a_3a_2^2 \\ &\quad + \frac{\omega_2}{\omega_3}\mu_5 a_3\sin\theta_6 + \frac{\alpha_8}{4\omega_3}a_3a_1^2\cos\gamma_9 \\ &\quad + \frac{\alpha_9}{4\omega_3}a_3a_2^2\cos\gamma_8 - \frac{Ha_1}{2\omega_3}\cos\gamma_7. \end{aligned} \qquad (32)$$

From equations (27-32), we have the following possible solutions:

Case 1: Squaring equations (27) and (28), then adding the squared results together gives the following frequency response equation:

$$\begin{aligned} &\left(\frac{3\alpha_1}{4\omega_1}\right)^2 a_1^6 + \left[\left(\frac{3\alpha_1}{4\omega_1^2}\right)(\alpha_2a_2^2 + \alpha_3a_3^2) - \frac{3\alpha_1\sigma_1}{2\omega_1}\right]a_1^4 \\ &+ \left[3\left(\frac{\alpha_2}{4\omega_1}\right)^2 a_2^4 + 3\left(\frac{\alpha_3}{4\omega_1}\right)^2 a_3^4 + \mu_1^2 + \sigma_1^2 + \left(\frac{\alpha_2\alpha_3}{2\omega_1^2}\right)a_2^2a_3^2 \right. \\ &\left. - \left(\frac{\sigma_1}{2\omega_1}\right)(\alpha_2a_2^2 + \alpha_3a_3^2) - \left(\frac{F}{2\omega_1}\right)^2\right]a_1^2 + \left[\left(\frac{2\omega_2}{\omega_1}\right)\mu_1\mu_4a_2\right]a_1 \\ &+ \left(\frac{\mu_4\omega_2a_2}{\omega_1}\right)^2. \end{aligned} \qquad (33)$$

Case 2: Squaring equations (29) and (30), then adding the squared results together gives the following frequency response equation:

$$\begin{aligned} &\left(\frac{3\alpha_4}{4\omega_2}\right)^2 a_2^6 + \left[\left(\frac{3\alpha_4}{4\omega_2^2}\right)(\alpha_5a_1^2 + \alpha_6a_3^2) - \frac{3\alpha_4\sigma_2}{2\omega_2}\right]a_2^4 \\ &+ \left[3\left(\frac{\alpha_5}{4\omega_2}\right)^2 a_1^4 + 3\left(\frac{\alpha_6}{4\omega_2}\right)^2 a_3^4 + \mu_2^2 + \sigma_2^2 \right. \\ &\left. + \left(\frac{\alpha_5\alpha_6}{2\omega_2^2}\right)a_1^2a_3^2 - \left(\frac{\sigma_1}{2\omega_2}\right)(\alpha_5a_1^2 + \alpha_6a_3^2) - \left(\frac{G}{2\omega_2}\right)^2\right]a_2^2 \\ &+ \left[\left(\frac{2\mu_2}{\omega_2}\right)(\mu_4\omega_1a_1 - \mu_5\omega_3a_3)\right]a_2 + \left(\frac{\mu_4\omega_1a_1}{\omega_2}\right)^2 \\ &+ \left(\frac{\mu_5\omega_3a_3}{\omega_2}\right)^2 - \left(\frac{2\omega_1\omega_3\mu_4\mu_5a_1a_3}{\omega_2^2}\right). \end{aligned} \qquad (34)$$

Case 3: Squaring equations (31) and (32), then adding the squared results together, gives the following frequency response equation:

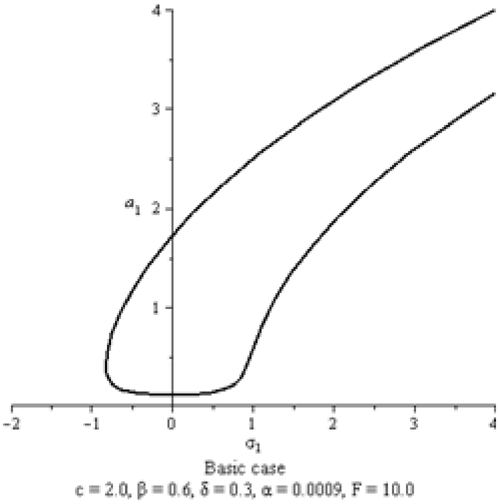
$$\left(\frac{3\alpha_7}{4\omega_3}\right)^2 a_3^6+\left[\left(\frac{3\alpha_7}{4\omega_3^2}\right)\left(\alpha_8 a_1^2+\alpha_9 a_2^2\right)-\frac{3\alpha_7\sigma_3}{2\omega_3}\right]a_3^4$$
$$+\Bigg[3\left(\frac{\alpha_8}{4\omega_3}\right)^2 a_1^4+3\left(\frac{\alpha_9}{4\omega_3}\right)^2 a_2^4+\mu_3^2+\sigma_3^2$$
$$+\left(\frac{\alpha_8\alpha_9}{2\omega_3^2}\right)a_1^2a_2^2-\left(\frac{\sigma_3}{2\omega_3}\right)(\alpha_8 a_1^2+\alpha_9 a_3^2)-\left(\frac{H}{2\omega_3}\right)^2\Bigg]a_3^2$$
$$+\left[\left(\frac{2\omega_2}{\omega_3}\right)\mu_3\mu_5 a_2\right]a_3+\left(\frac{\mu_5\omega_2 a_2}{\omega_3}\right)^2. \qquad (35)$$

4. Frequency Response Solutions

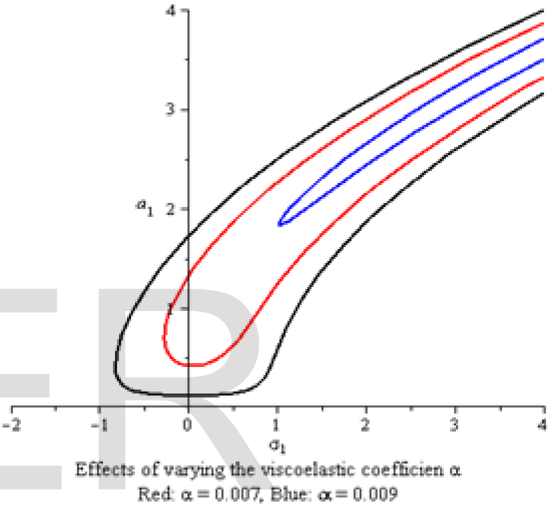
The periodic solutions corresponding to the fixed points of equations (21-26) for simultaneous internal and principle parametric resonances of the three modes are obtained when $a'_{1,2,3}=0$ and $\gamma'_{1,4,7}=0$. From the resulting equations, the frequency response equations (33-35) are obtained and solved numerically. The numerical results are presented Figs. (1-3) as the amplitudes $a_{1,2,3}$ against the detuning parameters $\sigma_{1,2,3}$ for different values of the system parameters.

4.1 Response curves of case 1 and 2:

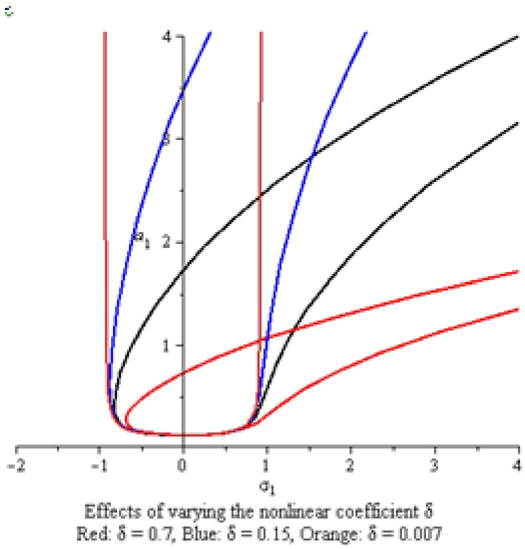
Considering Fig. (1a) as a basic case for comparison, it can be seen from Figs. (1b) and (1c) that as the viscoelastic coefficient α and the nonlinear coefficient δ increase, the steady state amplitude of the first mode a_1 decreases. More increase in δ leads the curves to be bent to the right indicating strong hardening nonlinearity effect. Whereas more decrease in δ can eliminate the possibility of the appearance of jumps. It can be seen from Fig. (1d) that as the axial moving speed c increases, the curves are shifted upward without any change in the magnitude of the amplitude a_1. While Fig. (1e) shows that as the natural frequency ω_1 by increasing the flexural stiffness β, the branches of the response curves converge to each other and the region of unstable solutions decreases. Figure (1f) indicates that as the parametric excitation F increases, the amplitude a_1 is increased. The response curves in Fig. (1g) are shifted upward with small increase in the amplitude a_1 as the steady-state amplitude of the second mode a_2 is increased. Whereas, the resonant response curves in Fig. (1h) are shifted to right as the steady-state amplitude of the third mode a_3 is increased.



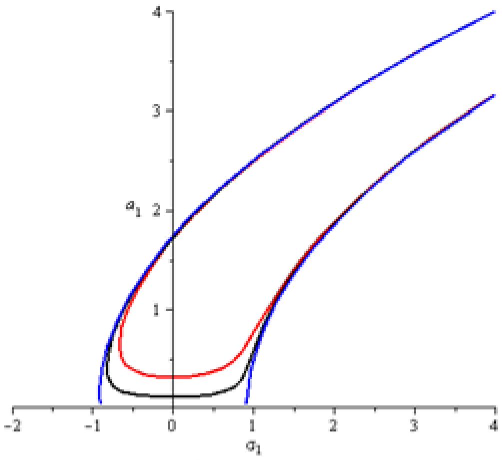
a)



b)

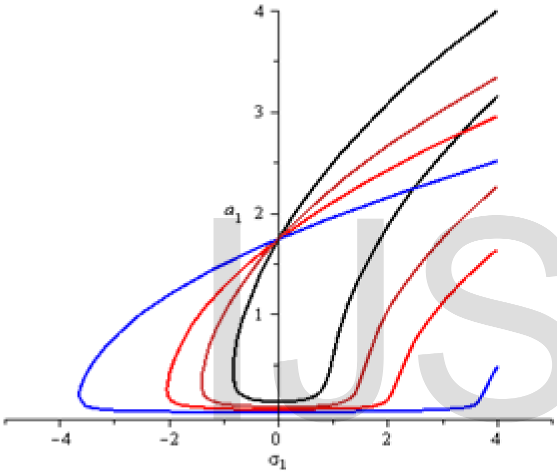


c)



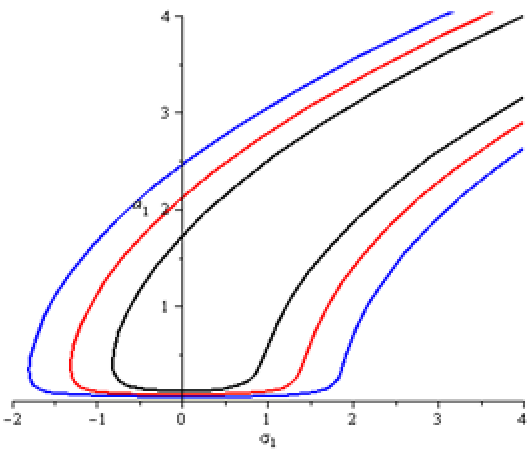
Effects of varying the axial moving speed c
Red: c = 5.0, Blue: c = 0.2

d)



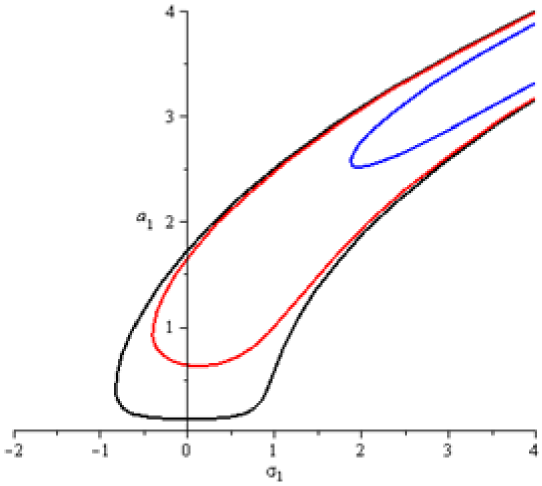
Effects of varying the natural frequency
Red: ω_1 = 2.3, Blue: ω_1 = 1.34, Orange: ω_1 = 3.34

e)



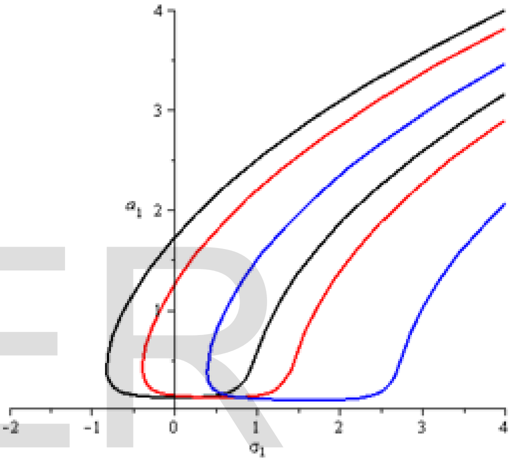
Effects of varying the excitation amplitude F
Red: F = 15.0, Blue: F = 20.0

f)



Effects of varying steady-sate amplitude of the second mode
Red, Blue: a_2 = 0.05, 0.2

g)



Effects of varying steady-sate amplitude of the third mode
Red, Blue: a_3 = 1.5, 1.8

h)

Fig. 1 Resonant frequency response curves for the first mode of the system, where a_2 = 0.01 and a_3 = 0.02.

For the second case, the steady-state amplitude a_2 is plotted against the detuning parameter σ_2, as shown in Fig. (2). Figures (2b), (2c), (2e)-(2h) illustrate similar effects of the system parameters to those reported in case 1. But in Fig. (2d), significant increase is noticed in the axial moving speed c as the steady-state amplitude a_2 is increased.

4.2 Response curves of case 3:

Different frequency response curves for the third mode of vibration are shown in Fig. (3), where effects of the parameters illustrate similar effects to those in case 1 except Fig. (3d), which shows that the steady-state amplitude a_3 decreases as the axial moving speed c is increased.

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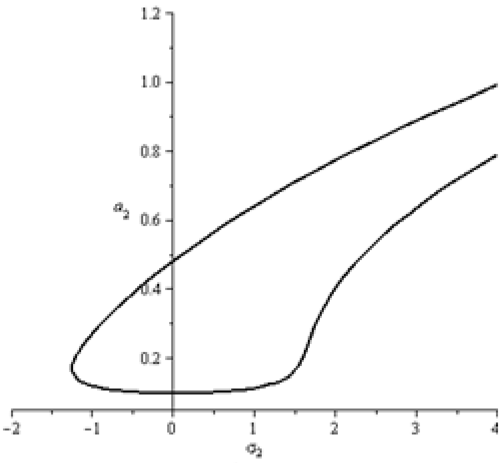
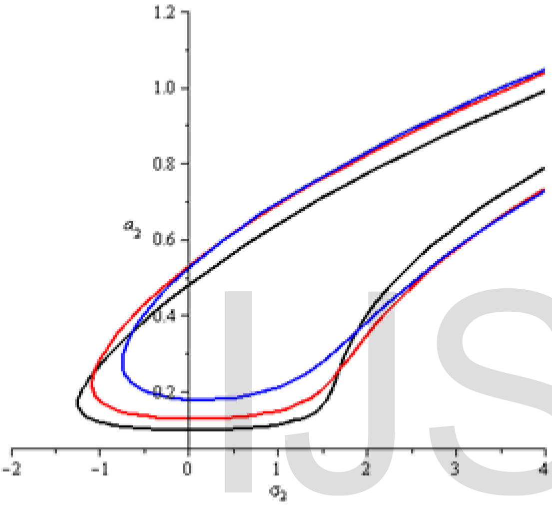


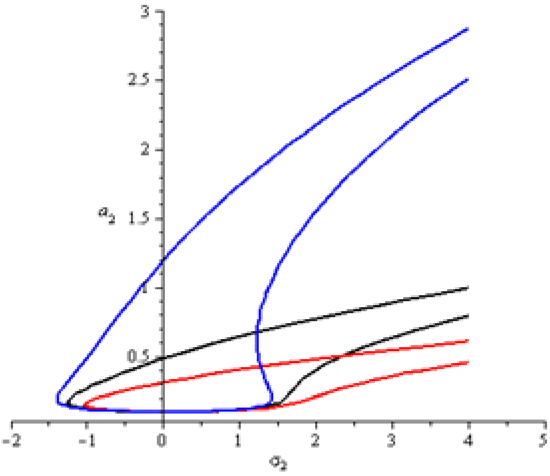
Basic case
$c = 2.0$, $\beta = 0.6$, $\delta = 0.3$, $\alpha = 0.0009$, $G = 15.0$

a)



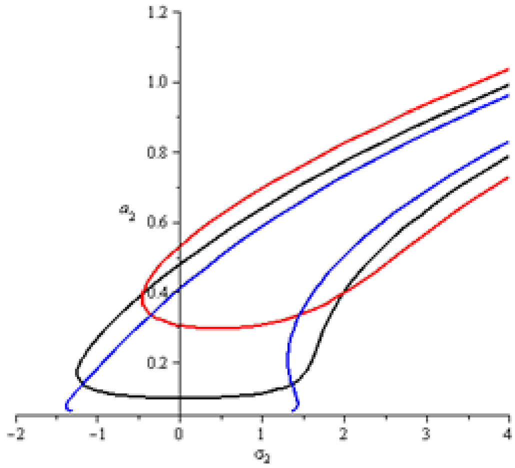
Effects of varying the viscoelastic coefficient α
Red: $\alpha = 0.0005$, Blue: $\alpha = 0.001$

b)



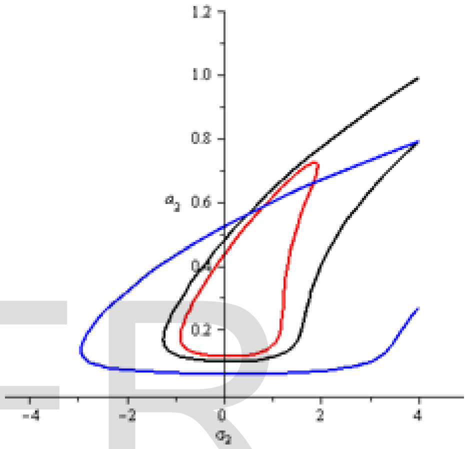
Effects of varying the nonlinear coefficient δ
Red: $\delta = 0.5$, Blue: $\delta = 0.1$

c)



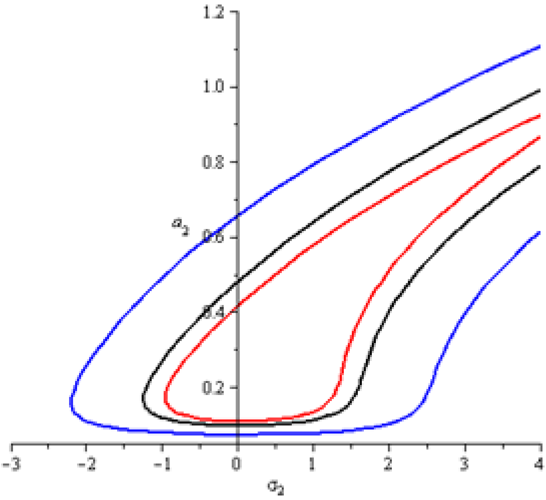
Effects of varying the axial moving speed c
Red: $c = 6.0$, Blue: $c = 0.8$

d)



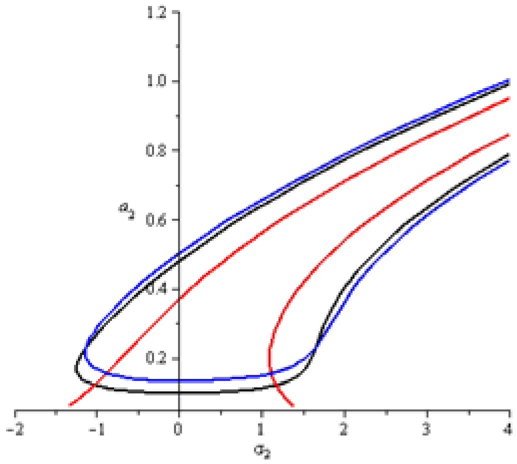
Effects of varying the natural frequency
Red: $\omega_2 = 7.0$, Blue: $\omega_2 = 2.36$

e)



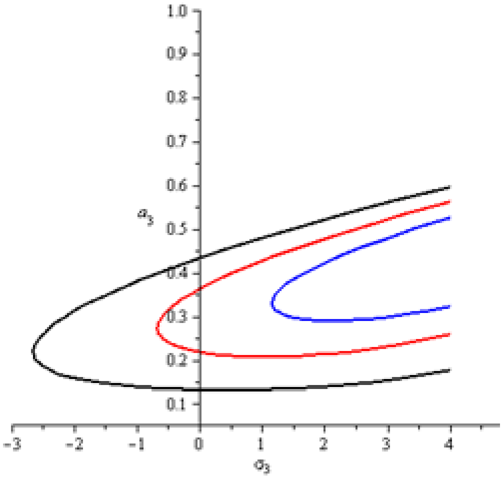
Effects of varying the excitation amplitude G
Red: $G = 12.0$, Blue: $G = 25.0$

f)



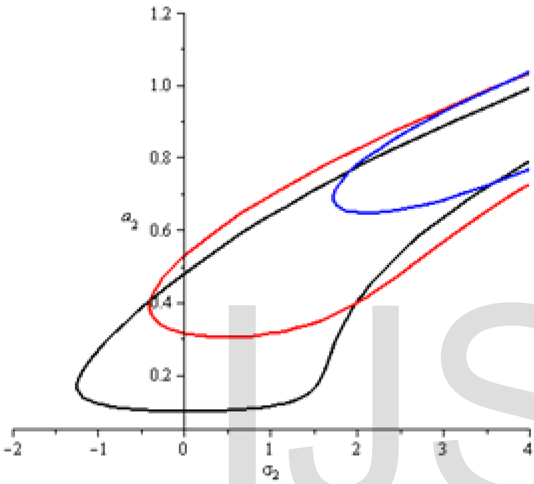
Effects of varying steady-sate amplitude of the first mode
Red, Blue: a_1 = 0.03, 0.001

g)



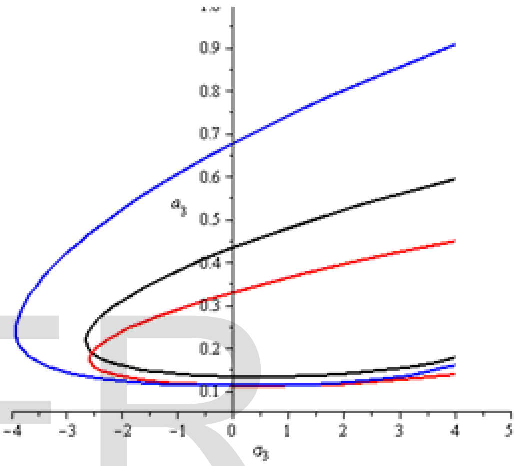
Effects of varying the viscoelastic coefficient α
Red: α = 0.0003, Blue: α = 0.0004

b)



Effects of varying steady-sate amplitude of the third mode
Red, Blue: a_3 = 0.05, 0.1

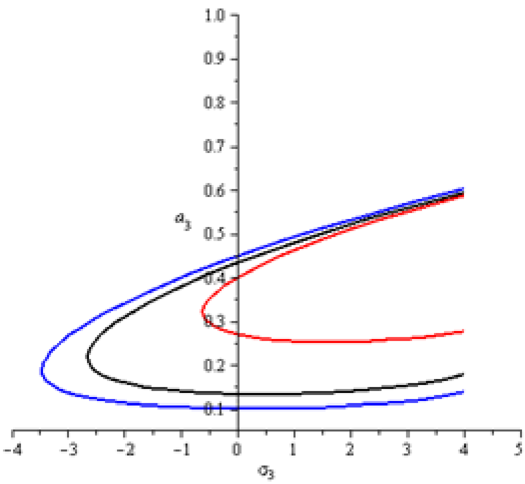
h)



Effects of varying the nonlinear coefficient δ
Red: δ = 0.4, Blue: δ = 0.2

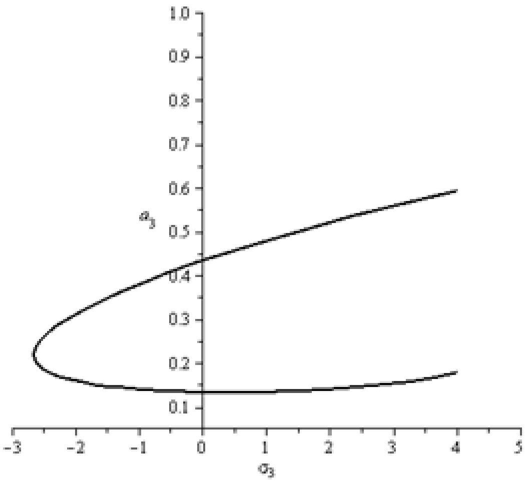
c)

Fig. 2 Resonant frequency response curves for the second mode of the system, where a_1 = 0.01 and a_3 = 0.02.



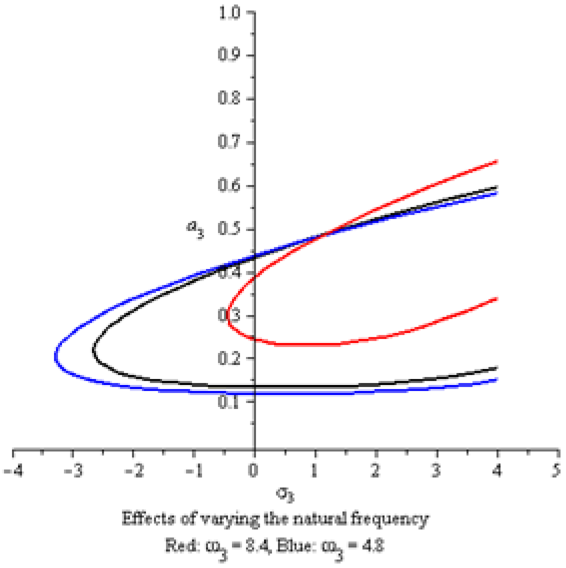
Effects of varying the axial moving speed c
Red: c = 4.5, Blue: c = 1.8

d)

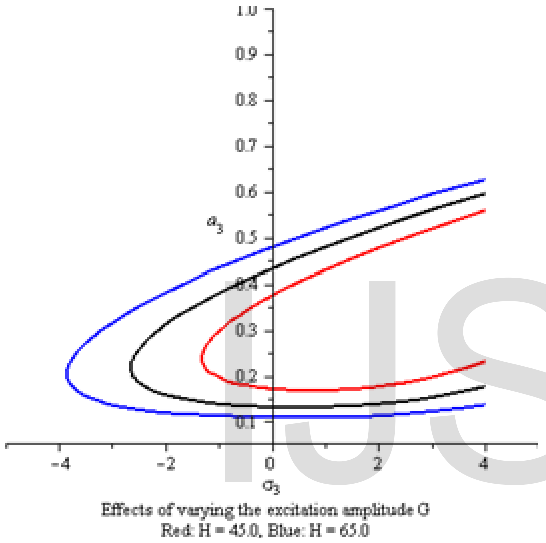


Basic case
c = 2.0, β = 0.6, δ = 0.3, α = 0.0001, H = 45.0

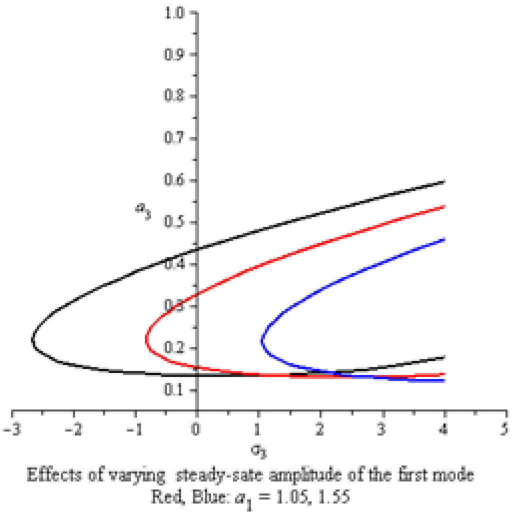
a)



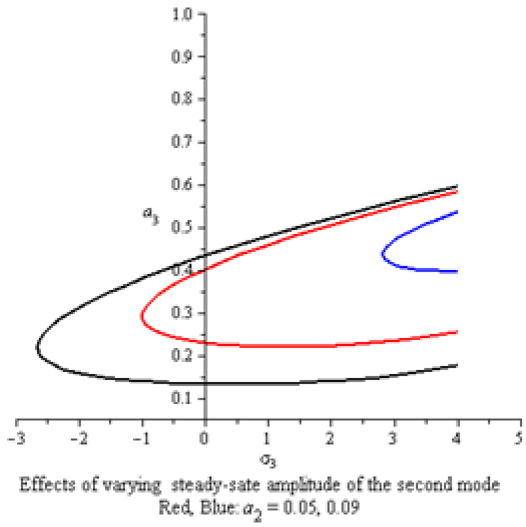
e)



f)



g)



h)

Fig. 1 Resonant frequency response curves for the third mode of the system, where a_1 = 0.05 and a_2 = 0.03.

5. Conclusion

The steady-state response of a three degree of freedom viscoelastic beam under parametric excitations is investigated, where internal as well as external resonance conditions have been considered. The method of multiple scales is applied to determine the simultaneous primary internal and parametric external resonance case and to study the effects of different parameters on system response. Three cases for the frequency response curves have been presented and investigated to illustrate the effects of viscoelastic coefficient, nonlinear coefficient, axial moving speed, the flexural stiffness, parametric forcing amplitudes and the steady-state amplitudes of the three modes of vibration. The following may be concluded:

1. The system has various interesting phenomenon such as jumps, multi-valued solutions, and hardening nonlinearities.
2. The steady-state amplitudes $a_{1,2,3}$ of the three modes are monotonic decreasing functions in the viscoelastic coefficient α.
3. The steady-state amplitudes $a_{1,2,3}$ are monotonic decreasing functions in the nonlinear coefficient δ. More decrease in δ may eliminate the possibility of the appearance of jumps.
4. The axial moving speed c affects the steady-state response in various ways. For the first mode, it shifts the frequency response curves upward, while as c increases, steady-state amplitude of the second mode is increased but the stead-state amplitude of the third mode is decreased.
5. The steady-state amplitudes $a_{1,2,3}$ are monotonic decreasing functions in the flexural stiffness β and the natural frequencies $\omega_{1,2,3}$.
6. The steady-state amplitudes $a_{1,2,3}$ are monotonic increasing functions in the parametric forcing amplitudes F, G and H.

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